DISENTANGLING THE RESEARCH LITERATURE ON "NUMBER SENSE": THREE CONSTRUCTS, ONE NAME

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We review research literature concerning "number sense" from several related fields. Whereas other authors have pointed to difficulty defining "number sense" or to some degree of inconsistency in the literature, we argue instead that this is a case of polysemy: There are 3 different constructs that go by the same name. In this article, we clarify the research literature concerning "number sense" by naming and defining these 3 constructs, identifying similarities and differences between them, and contrasting themes in each body of literature by drawing upon a sample of 124 research articles that focus on "number sense."

Keywords: Number Concepts and Operations

> What's in a name? That which we call a rose, by any other name, would smell as sweet. — Juliet in Romeo & Juliet, William Shakespeare

There has been increasing interest in "number sense" from researchers in fields including experimental psychology, mathematical cognition, special education, and mathematics education. As an illustration, in a search for research articles with "number sense" in the title, we found 13 articles published in the 1990s, 40 articles published in 2000–2009, and 71 articles published in 2010–2016. Yet there seem to be a wide variety of uses of the term *number sense*. For example, consider the following two article titles: "Relationships among computational performance, pictorial representation, symbolic representation and number sense of sixth-grade students in Taiwan" (Yang & Huang, 2004) and "Wild number sense in brood parasitic Brown-headed Cowbirds" (Low, Burns, & Hauber, 2009). As this contrasting pair illustrates, we find a wide range of uses of the term *number sense* in the research literature.

More broadly, researchers in the social and behavioral sciences have become concerned with impediments to progress resulting from confusion over constructs (Brown, 2015; Gintis, 2007; Larsen, Voronovich, Cook, & Pedro, 2013; Le, Schmidt, Harter, & Lauver, 2010; Shaffer, DeGeest, & Li, 2016). Two particular issues have the potential to plague the research literature: synonymy and polysemy. *Synonymy* refers to different terms having the same meaning. *Polysemy* refers to the same term being used in different ways. Larsen et al. (2013) argue that these issues result in a proliferation of constructs and meanings, leading to "reverse progress" (p. 1532) as less is known over time about relationships between constructs, relative to the number of constructs that appear in the literature. As we will demonstrate, we regard the varied uses of the term *number sense* in the research literature as a problematic case of polysemy.

Many authors have noted difficulties defining "number sense" or disparities in the definitions and descriptions found in the literature (e.g., Andrews & Sayers, 2015; Berch, 2005; Dunphy, 2007; Howell & Kemp, 2005; 2009; 2010; Lago & DiPerna, 2010). In a seminal article published 25 years ago, McIntosh, Reys, and Reys (1992) emphasized the need to clarify the meaning of "number sense" in order for related research to progress. More recent articles have pointed to differences in definitions and assumptions about "number sense" but have assumed that these reflect different "perspectives" concerning a single construct, rather than fundamentally different constructs with the same name (Berch, 2005; Andrews & Sayers, 2015).

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The above critiques of the "number sense" literature have each been situated within a particular field. Our review, which is a product of collaboration between researchers in special education and mathematics education, is concerned with the need for greater clarity in the "number sense" literature across fields. In this article, we categorize the literature focusing on "number sense" based on researchers' definitions and assumptions. Our systematic review of a sample of 124 research articles leads to a clear conclusion that is responsive to the issues identified above: Instead of disagreement over a single construct, we find three distinct "number sense" constructs at play in the literature. We argue that this is a problematic case of polysemy and a microcosm of broader issues of construct confusion in the social and behavioral sciences. It is difficult for research concerning a particular "number sense" construct to advance so long as authors continue to attempt to draw upon literature concerning different constructs that go by the same name. These different constructs involve contrasting assumptions about the nature of "number sense" and they are embedded in traditions with distinct orientations and concerns.

Method

We conducted a literature review to answer the following research question: How is the term *number sense* used in research literature in the social and behavioral sciences? In particular, how is "number sense" defined, and what assumptions do researchers make about the nature of "number sense"? We searched five databases (Academic Search Complete, Education FullText, ERIC, JSTOR, and Psyc INFO) for research articles with "number sense" in their titles. It was important to control the scope of our review in this way, because we sought to identify how "number sense" was defined and analyzed and to identify characteristics of "number sense" research. This being our purpose, including all articles that made any mention of "number sense" would have muddied our results. The numbers of journal articles that mentioned "number sense" varied from 168 to 1,587 in the databases listed above, and many of these were not research studies or were studies that did not focus on "number sense." For the purposes of our review, the relevant studies were those in which "number sense" was central to the research. We found the inclusion of "number sense" in the title of the article to be a reasonable proxy for this centrality.

We searched for all such articles that met the criteria described above and that were published on or before December 31, 2016. We focused on research articles published in peer-reviewed journals. Thus, we filtered out practitioner articles, books and book reviews, and conference abstracts and proceedings. We also filtered out publications not written in English. A final count of 124 articles qualified for inclusion in our sample. We recognize that some high-quality articles concerning "number sense" may not be included in our sample as a result of these requirements. Our purpose was not to provide comprehensive reviews of the literature belonging to each "number sense" tradition. It was to identify and describe "number sense" constructs based on a sufficient sample of the research literature associated with each construct.

In addition to the sample of articles described above, we consulted seminal works and publications of historical significance that explicitly addressed "number sense." We identified these based on their being cited frequently in our sample of research articles and/or representing a synthesis of research related to "number sense" in a particular research tradition. We included in this category the works of Dehaene (1997/2011), Geary, Berch, and Koepke (2015), Sowder and Schappelle (1989), and Sowder (1992). Consulting these sources provided us greater access and insights into the history of "number sense" research and enabled us to answer questions concerning definitions, assumptions, findings, and themes in cases in which a consensus could not be identified within our sample of articles.

We initially read a selection articles from our sample, focusing on authors' interpretations of "number sense" and the apparent origins of those interpretations (based on citations and use of key constructs), as well as the populations studied and methods used. We proceeded using open coding to define distinct "number sense" constructs. We refined our definitions through constant comparative analysis as we reviewed additional articles (Corbin & Strauss, 2008). Once we had reached a saturation point, we settled on three constructs: (a) *innate number sense*, (b) *early number sense*, and (c) *mature number sense*.

We grouped the bodies of literature related to each of the three constructs into distinct *research traditions*. We each took primary responsibility for reading and summarizing the literature belonging to one of the traditions. We developed these summaries iteratively with feedback from one another. We then compared our summaries based on key concepts, analytic approaches, findings, and themes. Questions that arose concerning similarities and differences between traditions or lack of clarity regarding any terms led us to return to our sample of articles and/or our set of seminal publications for answers. This process, too, was iterative. We refined our summaries of the aspects of each tradition in order to focus more clearly on the similarities and differences between traditions. Length constraints for this manuscript do not permit us to report on the research traditions in any detail; however, the themes that we identified helped to focus our descriptions of the "number sense" constructs and related concepts from each tradition.

Findings: Three "Number Sense" Constructs

We describe the three "number sense" constructs, along with key concepts related to each of them, based on our review of the literature. We also highlight similarities and differences in assumptions that distinguish these constructs.

Overview of the Three "Number Sense" Constructs

Innate number sense (INS) is believed to be an inborn set of neurological abilities that is common to humans and some animals. Thus, INS research involves infants, children, adults, and non-human animals (e.g., Libertus & Brannon, 2009; Halberda & Feigerson, 2008; Low et al., 2009). This construct concerns perception and discrimination of magnitudes, rather than explicit knowledge of number words or symbols. Much of the research with humans involves observing brain activity while participants perform tasks such as determining which of two sets consists of more items (e.g., Dehaene, 2001; Libertus & Brannon, 2009; Stoianov & Zorzo, 2012). Dehaene (1997/2011) believes that most people are born with an equal endowment of number sense and, therefore, INS is not predictive of success in learning mathematics. Dehaene's use of the term *the* number *sense* (with the definite article *the* and emphasis on the word *number*) is indicative of the view of INS as an innate sense, which is related to visual and auditory perception.

Early number sense (ENS), in contrast to INS, includes learned skills that involve explicit number knowledge, such as counting items using number words and comparing numbers represented symbolically as numerals. Some researchers believe that ENS builds upon the more basic INS (Andrew & Sayers, 2015; Aunio et al., 2005; Geary, et al., 2015). Levels of ENS skills vary from person to person and are influenced by education and experiences in early childhood (Cheung & McBride-Chang, 2015; Dunphy, 2006). ENS is regarded as an important predictor of success in school mathematics (Dyson, Jordan, & Glutting, 2011; Locuniak & Jordan, 2008; Jordan, Kaplan, Locuniak, & Ramineni, 2007). Accordingly, ENS skills are well aligned with school mathematics, especially in the early childhood years (preschool to Grade 2). Typically, studies of ENS involve young children or students with disabilities. ENS research does not

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belong to a single field. It is conducted primarily by researchers in mathematics education, special education, and cognitive psychology.

We use the term *mature number sense* (MNS) to distinguish the "number sense" construct that features prominently in the mathematics education research literature. MNS encompasses multidigit and rational number sense, and studies focus primarily on middle-grades (i.e., upper elementary and middle school) students and preservice teachers. Like ENS, the MNS construct refers to something learned, rather than innate. In contrast to ENS, MNS is typically described in terms of *components*, which refer to conceptual structures and habits of mind, rather than skills (e.g., McIntosh et al., 1992; Reys & Yang, 1998). For example, MNS is associated with flexibility in mental computation (Markovits & Sowder, 1994). Furthermore, whereas ENS is well aligned with school mathematics, MNS is often contrasted with school mathematics: Students who competently perform computations using standard algorithms may not exhibit characteristics of MNS, such as flexibility (Reys & Yang, 1998; Reys et al., 1999).

Key Concepts in "Number Sense" Research Traditions

Having provided an overview of the three constructs, we delve deeper into related concepts that appear in the three corresponding research traditions.

Key concepts in INS research. According to Dehaene (2001), "Number sense is a short-hand for our ability to quickly understand, approximate, and manipulate numerical quantities" (p.16). INS is considered part of an evolutionary process related to neurological abilities. Specifically, three neurological abilities are associated with INS: perceptual subitization, magnitude discrimination, and the use of a mental number line. *Perceptual subitization* can be defined as rapidly or immediately identifying numerosities of sets consisting of up to three or four items (Clements, 1999; Dehaene, 2001). Any numbers that are beyond four are then approximated with less precision (Clark & Grossman, 2007). *Magnitude discrimination* consists of indicating the difference in cardinality between two sets of items (presented visually or auditorily) (Dehaene, 2001). The *mental number line* is a mental approximation of magnitude based on a continuous number line believed to be present in an individual's mind (Clark & Grossman, 2007). The use of a mental number is inferred from the ability to "quickly decide that 9 is larger than 5, that 3 falls in the middle of 2 and 4, or that 12 + 15 cannot equal 96" (Dehaene, 2001, p. 16). (Although comparisons of numerals require explicit number knowledge, they are taken as evidence of a mental number line in humans who have developed such knowledge.)

Key concepts in ENS research. ENS is conceptualized and studied as a set of skills. In a prominent example, Jordan and colleagues (2006) focused on "assessed skills that have been validated by research and are relevant to the mathematics curriculum in primary school" (p. 154). Six main skills are focal in ENS research: number recognition, counting, number patterns, number comparison, number operations, and estimation. *Number recognition* requires children to associate the number symbols with the vocabulary and meaning of numbers. *Counting* includes ordinality, cardinality, and counting backward or forward starting with an arbitrary number. *Number patterns* is the ability to copy a given pattern or identify a missing number in a sequence. *Number comparison* refers to awareness of the magnitude of given numbers and the ability to make comparisons between different magnitudes. *Number operations* involves the ability to perform simple calculations of sums and differences within 10 or 20. *Estimation* refers to magnitude estimation of symbolic and non-symbolic quantities, including the use of a number line to identify the approximate location of a number (Andrews & Sayers, 2015; Berch, 2005;

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Baroody et al., 2012; Ivrendi, 2011; Jordan et al., 2006; Howell & Kemp, 2010; Malofeeva, Day, Saco, Young, & Ciancio, 2004; McGuire, Kinzie, & Berch, 2012).

Key concepts in MNS research. McIntosh and colleagues (1992) provided a definition of MNS that has often been cited or paraphrased:

> Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity. (p. 3)

The above description seems to capture the gist of the term *number sense*, as it is commonly used in the mathematics education community. It focuses on habits of mind and ways of behaving mathematically that are considered desirable, such as flexibly manipulating numbers.

Despite this holistic definition, MNS is typically partitioned into up to six components: *understanding of the meaning and size of numbers* (e.g., to compare fractions), *understanding and use of equivalent representations of numbers* (e.g., to write rational numbers in different ways), *understanding the meaning and effect of operations* (e.g., to reason about the effect of dividing by a number between 0 and 1), *understanding and use of equivalent expressions* (e.g., to compare expressions involving different numbers and/or operations), *flexible computing and counting strategies for mental computation, written computation, and calculator use* (e.g., to select strategies and perform mental computation), *and measurement benchmarks* (e.g., to estimate the height of an object) (Reys et al., 1999; p. 62). Many studies use assessments designed to measure specified components of number sense (e.g., Yang & Lin, 2015).

Similarities and Differences

In summary, INS is regarded as innate and equally distributed among normal people at birth. It is also found in some animals. INS consists of a set of basic neurological abilities, which do not account for success in learning mathematics. ENS, by contrast, is regarded as learned. It is unequally distributed among people and is not found in animals. ENS is typically conceptualized as consisting of a set of skills, and these skills are well aligned with primary-grades mathematics. MNS is also learned and unequally distributed among people. It is typically described as consisting of a set of components, which include conceptual understandings and habits of mind. In contrast to ENS, MNS is often described as being at odds with students' typical experiences in school mathematics and the mathematical knowledge and orientation that result.

Discussion and Conclusion

Whereas other authors have observed differences in definitions or interpretations of "number sense," they have assumed that this confusion surrounds a single construct (Andrews & Sayers, 2015; Berch, 2005). For example, Berch (2005) referred to "*the* concept of number sense" (p. 333, emphasis added) and Andrews and Sayers (2015) described "number sense" as "*a* poorly-defined construct" (p. 257, emphasis added). These previous observations were made from the perspective of researchers in the ENS tradition, and their purpose was not to clarify the "number sense" literature more broadly. Whereas INS and MNS research represent two extremes, ENS research lies in between them, and particular studies may lean closer to one side or the other. Thus, it is ENS researchers who face the most potential for confusion in attempting to navigate the muddled "number sense" literature. It is no surprise, then, that ENS researchers have taken

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the lead in recent efforts to clarify what "number sense" means in order to facilitate progress in ENS research.

Our systematic review led to the identification of three "number sense" constructs. To the best of our knowledge, our review of the "number sense" literature is the first of its kind. It had the express purpose of analyzing the use of the term *number sense* in research literature across fields in order to disentangle the meanings and assumptions associated with the term. The contrasting features of the constructs underscore the need to clearly distinguish them.

The results presented above contribute to the literature by clarifying distinctions between three constructs that have gone by the same name. The widespread use of the term *number sense* to refer to three distinct constructs belonging to different traditions has led to some confusion and has not gone unnoticed in the literature. By systematically coding the articles in our data set according to the authors' assumptions about "number sense" and their methods of investigation, we were able to clarify the nature of the construct within each tradition.

The differences that we identified in definitions and assumptions are not a trivial observation about the literature. We noted in our review many instances of inappropriate citations across research traditions. For example, some MNS articles cite INS research to support their claims about "number sense" despite the fact that their research concerns a different construct. To be clear, authors working in one "number sense" tradition should not be citing authors working in another tradition, unless there is explicit acknowledgment of the differences between traditions and unless there is a particular reason for the citation. We suggest that researchers use terms such as INS, ENS, or MNS to distinguish the "number sense" construct that they are investigating. Although "number sense" is a catchy term that rolls off the tongue, its loose usage across related research traditions has led to confusion and impediments to progress.

In conclusion, we find in the "number sense" literature a problematic case of polysemy. As Larsen et al. (2013) state, "The task of integrating research by connecting synonymous constructs and parsing polysemous constructs is an urgent one if behavioral science is to advance" (p. 1533). We agree, and we add that the same issue applies to research in the social sciences. In particular, in order to propel progress in "number sense" research in a variety of fields, there is a need to clarify the construct under investigation within each tradition.

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